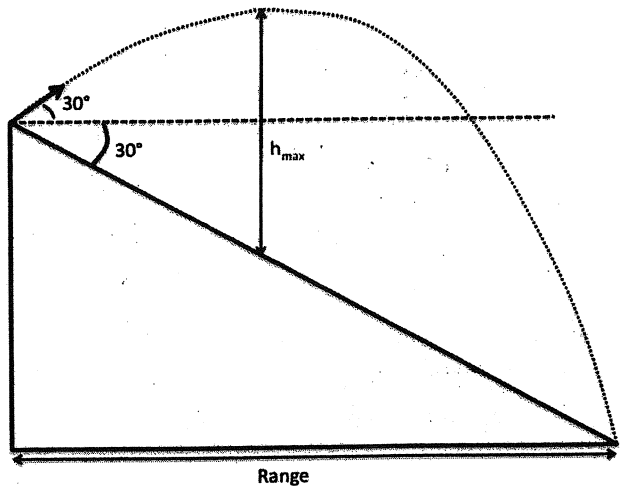
Final Exam 2048 H

Name:

- There are five questions. Please make sure that you have all the questions before you begin.
- Please make it clear for me to see how you are thinking.
- Exams should be graded by Friday morning. You can pick up the exam as well as its solution from me. I will be in either 149 or 420 MAP.

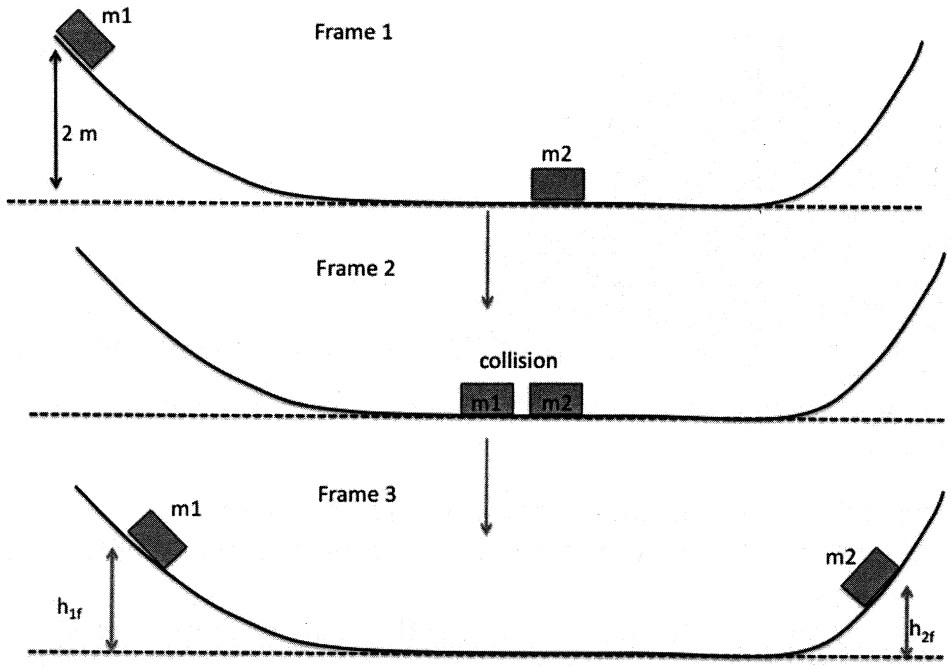
Problem 1 [15 pts]



A projectile, with mass 10 kg, is launched with 30 degrees to horizontal on the slope, which is angled also at 30 degrees as depicted above. Initial speed of the projectile is 20 m/s. Gravity (9.8 m/s^2) points downwards on the figure as usual. Neglect consideration of air resistance.

a) Define t_f as the time at which the projectile lands on the slope. Write down the mathematical relationship between $x(t_f)$ and $y(t_f)$ (these being x and y positions of the projectile)? [2 pts]
b) Calculate the range [4 pts]
c) Calculate h_{max} [4 pts]
d) Calculate the acceleration of the projectile at h_{max} [2 pts]
e) Calculate the speed of the projectile at landing [3 pts]

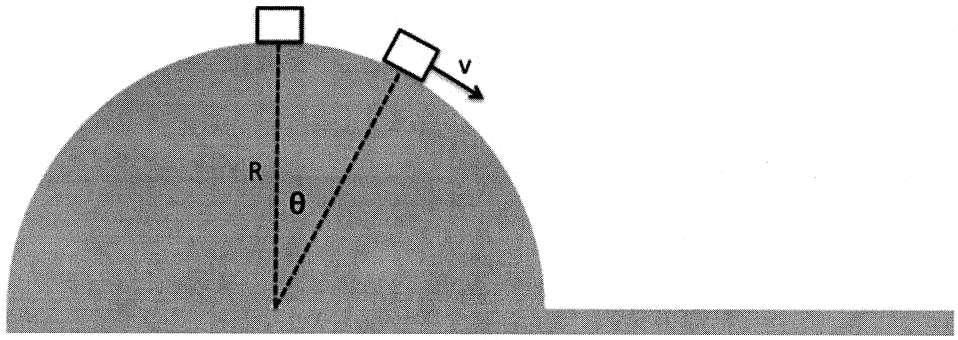
Problem 2 [15 pts]



Three different frames above describe a sequence of events. In frame 1, objects start from rest. M1 accelerates down the slope and elastically collides with m2 in the frame 2. After the collision, both masses eventually each reach attain some maximum height. This question is about these sequences. m1 = 1 kg and m2 = 4 kg.

a) Find the velocity of m1 as it is about to hit m2 [2 pts]
b) Masses collide elastically, what are the conserved quantities in the collision? [2pts]
c) What are the velocities of m1 and m2 after the collision [7 pts]
d) Find the heights indicated in the frame 3. Note that the objects may attain the maximum heights at different times. Frame 3 is only there as a guide. [4 pts]

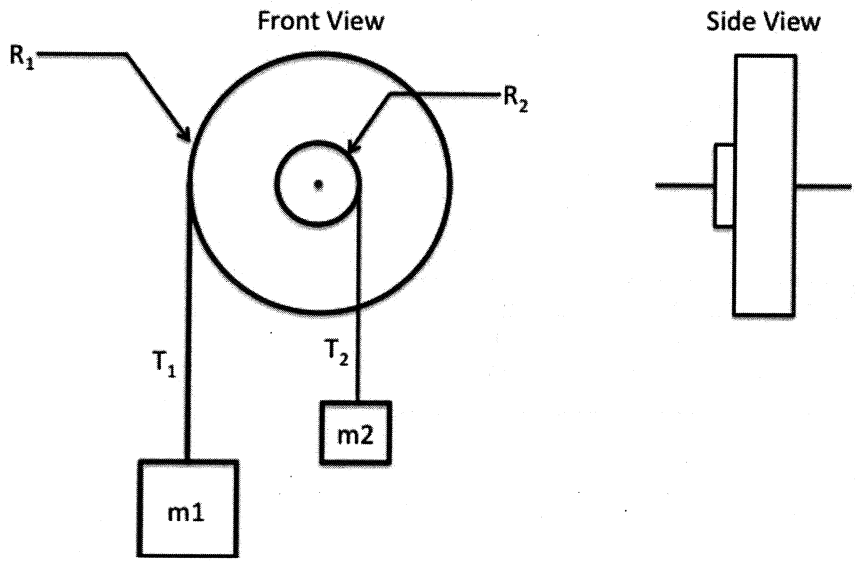
Problem 3 [25 pts]



A mass with m=5 kg slides down a hemispherical shaped hill with a radius (R) of 1 m. There are no friction.

a) Draw the force diagram of the mass when it is at angle θ. [4 pts]
b) Calculate the normal force exerted on the mass by the hemisphere as a function of θ. [4 pts]
c) What is the condition for it to stay in contact with the hemisphere? [4 pts]
d) When is the critical angle, θ_{max}, after which the object is no longer in contact with the hemisphere. [9 pts]
e) What is the critical angle if the object starts sliding from initial angle of 30°? [4 pts]

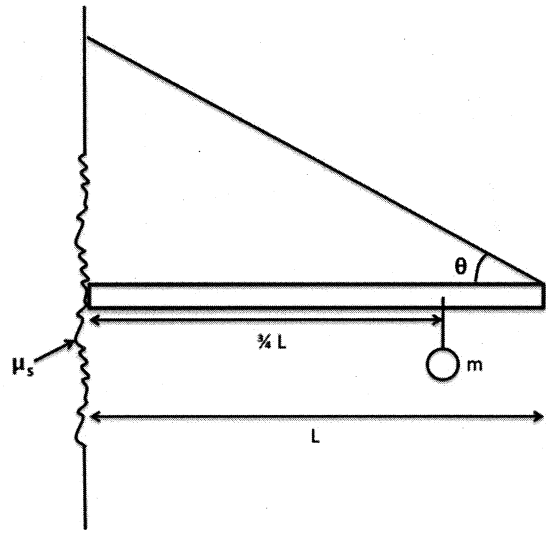
Problem 4 [20 pts]



Two massive pulleys are glued together and set up with two masses. R_1=3m, R_2=1m, M_1=10kg, and M_2=5kg. Pulley's mass is 100 kg and its moment of inertia is given by $\frac{1}{2}MR_1^2$ or $\frac{1}{2}(100)\times 3^2 = 450kg\times m^2$.

a) Draw force diagrams for m1, m2, and the pulley. [3 pts]
b) Are the tensions equal to each other and are the accelerations of m1 and m2 equal to each other? [2 pts]
c) Calculate the angular acceleration of the pulley [10 pts]
d) Calculate the tension(s) [5 pts]

Problem 5



A log with mass 10 kg is held on a wall horizontally with the rope on a wall with finite static friction. L = 1m. Small mass m is hanging from the log at ¾ L as drawin in the diagram. Hint: it is easier if you don't plug in the numbers until the end. I suggest keeping everything in T, θ, g, μ_s, M, and m, and then plug in the numbers.

a) Draw the force diagram for the log [1 pt]
b) Setting θ=30° and m=1 kg, calculate the minimum coefficient of static friction necessary to have the system in equilibrium. [8 pts]
c) With m=1 kg, what is the maximum angle θ possible to maintain the equilibrium with static friction at the wall? [For a normal materials the maximum coefficient of static friction is 1] [8 pts]
d) At θ=45°, what is the maximum mass, m, one can hang while maintaining static equilibrium. [For a normal materials the maximum coefficient of static friction is 1] [8 pts]